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A note on isometries of Lipschitz spaces. (English) Zbl 1341.46019

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This paper deals with spaces of vector-valued Lipschitz functions. Let $(V, \|\cdot\|)$ and $(W, \|\cdot\|)$ be real or complex Banach spaces and let X and Y be compact metric spaces. The spaces $\mathrm{Lip}(X, V)$ and $\mathrm{Lip}(Y, W)$ consist of all Lipschitz functions defined on X and Y, with values in V and W, respectively. These spaces are endowed with the norm $\|f\| = \max\{\mathcal{L}(f), \|f\|_\infty\}$, where $\mathcal{L}(f)$ is the Lipschitz constant of f. The map $T : \mathrm{Lip}(X, V) \to \mathrm{Lip}(Y, W)$ has property Q if, for every $y \in Y$ and $w \in W$, there exists $v \in V$ such that $T\hat{v}(y) = w$, where $\hat{v}$ denotes the constant map in $\mathrm{Lip}(X, V)$ equal v.

The statement of the main theorem in this paper generalizes a result in [*F. Botelho* et al., J. Math. Anal. Appl. 381, No. 2, 821–832 (2011; Zbl 1228.46020)] by removing a quasi-sub-reflexivity condition on the range spaces.

Theorem. Let X and Y be compact metric spaces and let V and W be Banach spaces with trivial centralizers. Let $T : \mathrm{Lip}(X, V) \to \mathrm{Lip}(Y, W)$ be a surjective linear isometry such that both T and T^{-1} have property Q, then T is a weighted composition operator of the form

$$Tf(y) = J(y)f(\phi(y))$$

for all $f \in \mathrm{Lip}(X, V)$ and $y \in Y$, where $\phi : Y \to X$ is a bi-Lipschitz homeomorphism and J is a Lipschitz map from Y into the space of surjective linear isometries from V into W.

The author raises the question of whether it is possible to relax the compactness assumption on the metric spaces.

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MSC:

46E15 Banach spaces of continuous, differentiable or analytic functions
46B04 Isometric theory of Banach spaces
46E40 Spaces of vector- and operator-valued functions
26A16 Lipschitz (Hölder) classes

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isometry; vector-valued Lipschitz function; Banach-Stone theorem

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