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An integral type characterization of constant functions on metric-measure spaces. (English)

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J. Math. Anal. Appl. 385, No. 1, 194-201 (2012).

Let (X,d,μ) be a metric measure space of doubling dimension k (i.e., $\mu(B(x,R)) \le C(R/r)^k \mu(B(x,r))$ uniformly in $R > r > 0$). Under a further technical postulate, the author shows that constant functions in domains $\Omega \subset X$ are characterized by the integrability condition

$$\int_\Omega \int_\Omega \frac{|f(x)-f(y)|^p}{d(x,y)^{k+p}} d\mu(x) d\mu(y) < \infty,$$

where $p \ge 1$. This extends a result of *H. Brezis* [Russ. Math. Surv. 57, No. 4, 693–708 (2002); translation from Usp. Mat. Nauk 57, No. 4, 59–74 (2002; Zbl 1072.46020)] for $X = \mathbb{R}^k$.

The technical postulate is as follows: Let u_n be Lipschitz functions in a ball $B \subset X$ with local Lipschitz constants denoted by $\mathrm{Lip} u_n$. If $\|\mathrm{Lip} u_n\|_1 \to 0$ and $\|u_n - u\|_1 \to 0$ for some $u \in L^1(B,\mu)$, then $u \equiv 0$ almost everywhere in B.

Reviewer: Tuomas Hytönen (Helsinki)

MSC:

46E35 Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems

46E30 Spaces of measurable functions (L^p-spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)

30L99 Analysis on metric spaces

Keywords:
doubling measure; Hölder continuity; metric measure space

Full Text: DOI

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