Ranjbar-Motlagh, Alireza
Generalized Rademacher-Stepanov type theorem and applications. (English) [Zbl 1180.58007]
Z. Anal. Anwend. 28, No. 3, 249-275 (2009).

The author generalizes a theorem of Stepanov which provides a necessary and sufficient condition for almost everywhere differentiability of functions over Euclidean spaces. Precisely, an L^p-type generalization of the Stepanov theorem is proved and it is extended to the settings of Orlicz spaces. An application of this generalized Rademacher-Stepanov type theorem is given to the Sobolev and bounded variation maps with values into a metric space. It is shown that several generalized differentiability type theorems are valid for Sobolev maps acting from a Lipschitz manifold into a metric space. As a byproduct, it is shown that the Sobolev spaces of Korevaar-Schoen and Reshetnyak are equivalent.

Reviewer: Dian K. Palagachev (Bari)

MSC:

58C20 Differentiation theory (Gateaux, Fréchet, etc.) on manifolds
46E30 Spaces of measurable functions (L^p-spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)

Cited in **1** Document

Keywords:

Rademacher and Stepanov theorems; Sobolev and bounded variation spaces; generalized differentiability; Lipschitz manifolds; Orlicz spaces

Full Text: DOI

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