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Generalized Stepanov type theorem with applications over metric-measure spaces. (English)

Zbl 1152.28004
Houston J. Math. 34, No. 2, 623-635 (2008).

Stepanov's differentiability theorem provides a necessary and sufficient condition for the Lebesgue almost everywhere differentiability of real-valued functions on $\mathbb{R}^n$. It is a generalized version of Rademacher's theorem, i.e., Lipschitz functions over finite-dimensional Euclidean spaces are differentiable almost everywhere.

The author proposes to extend an L^p-type generalization of Stepanov's differentiability theorem to metric measure spaces. He applies his generalized Stepanov type theorem to Sobolev functions and functions of bounded variation in order to get the L^p-type generalized differentiability for these functions. The proof of this generalized differentiability theorem is a combination of procedures taken by Campanato and Stepanov. As an application, an answer to the positive is given for a problem raised by Balogh, Rogovin, and Zuercher about L^p-type generalized differentiability of functions of bounded variation over metric measure spaces. Moreover, the author's results extend the main results of Bjoern and Balogh, Rogovin, and Zuercher.

Reviewer: Werner Strauß (Stuttgart)

MSC:

28A15	Abstract differentiation theory, differentiation of set functions
46E35	Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems
46G05	Derivatives of functions in infinite-dimensional spaces
49J52	Nonsmooth analysis

Keywords:

Stepanov's differentiability theorem; Sobolev functions; functions of bounded variation