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A remark on the Bourgain-Brezis-Mironescu characterization of constant functions. (English summary)

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The author presents a very simple proof of a generalization of the following:

Theorem 1. Let $f\colon \Omega \to \mathbf{R}$ be a Lebesgue measurable function on a domain $\Omega \subseteq \mathbf{R}^N$ such that

$$\int_{\Omega\times\Omega} \frac{|f(y)-f(x)|}{|y-x|^{N+1}}\,dydx < \infty.$$

Then f is constant.

In particular he proves the following:

Theorem 2. Suppose f and Ω are the same as in Theorem 1, and let $\omega\colon [0,+\infty) \to [0,+\infty)$ be a convex function such that $\omega(t)=0$ if and only if $t=0$. Then, if

$$\int_{\Omega\times\Omega} \omega\left(\frac{|f(y)-f(x)|}{|y-x|}\right)\frac{dy}{|y-x|^N}dx < \infty$$

holds, the function f is a constant. *Steven George Krantz*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.