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A note on isometries of Lipschitz spaces. (English summary)
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A function f from a metric space (X,d) to a Banach space (real or complex) V is said to be a *Lipschitz* function if

$$L(f)=\sup_{x\neq y}\frac{\|f(x)-f(y)\|}{d(x,y)}<\infty.$$

The number $L(f)$ is said to be the Lipschitz constant for f. For a compact metric space X, the space $\mathrm{Lip}(X,V)$ is the Banach space of all V-valued Lipschitz functions f on X with the norm $\|f\|=\max\{L(f),\|f\|_\infty\}$, where $\|f\|_\infty$ is the usual sup norm. The purpose of the paper under review is to generalize a characterization of isometries between such Lipschitz spaces given by F. Botelho, J. E. Jamison and the reviewer in [J. Math. Anal. Appl. **381** (2011), no. 2, 821–832; MR2802117]. The generalization removes the quasi-sub-reflexivity condition imposed on the Banach spaces.

For compact metric spaces X and Y and Banach spaces V and W, a linear map $T\colon \mathrm{Lip}(X,V)\to\mathrm{Lip}(Y,W)$ is said to have property Q if for $y\in Y$ and $w\in W$, there is a constant function F in $\mathrm{Lip}(X,V)$ such that $TF(y)=w$. The author proves that if T, as above, is a linear isometry such that both T and T^{-1} have property Q, then $\|Tf\|_\infty=\|f\|_\infty$ for every $f\in\mathrm{Lip}(X,V)$. Next it is shown that when X is compact, the elements of $\mathrm{Lip}(X,V)$ are dense in the continuous functions $C(X,V)$ with the sup norm. These two facts are now combined with a well-known version of the Banach-Stone Theorem to obtain the following main result: If X and Y are compact metric spaces, V and W are Banach spaces with trivial centralizers, and $T\colon\mathrm{Lip}(X,V)\to\mathrm{Lip}(Y,W)$ is a surjective linear isometry such that both T and T^{-1} have property Q, then T is a weighted composition operator of the form

$$Tf(y)=J(y)f(\phi(x)),$$

for all $f\in\mathrm{Lip}(X,V)$ and $y\in Y$, where $\phi\colon Y\to X$ is a bi-Lipschitz homeomorphism and J is a Lipschitz map from Y into the space of surjective linear isometries from V to W.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.