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Generalized Rademacher-Stepanov type theorem and applications. (English summary)

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A functional $A\colon \mathbb{R}^n \to \mathbb{R}$ is said to be an L^p-differential of $f\colon \mathbb{R}^n \to \mathbb{R}$ at a point x if

$$\lim_{r\to 0} r^{-n-p} \int_{B(x,r)} |f(y) - f(x) - A\cdot(y-x)|^p dx = 0.$$

The author proves that a sufficient condition for L^p-differentiability a.e. is that

$$\limsup_{r\to 0} r^{-n-p} \int_{B(x,r)} |f(y) - f(x)|^p dy < \infty$$

for a.e. $x \in \mathbb{R}^n$. A similar result is proved for mappings on open domains in Lipschitz manifolds, with values in a Banach space or in a metric space. As a consequence, it is shown that the Sobolev spaces of Korevaar-Schoen and of Reshetnyak are equivalent. Further, L^Φ-differentiability related to Orlicz spaces is introduced and a result of Rademacher-Stepanov type is applied to obtain (Trudinger type) exponential differentiability of $W^{1,n}$-functions. *Jan Malý*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.