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A note on the Poincaré inequality. (English summary)

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Let (X, d, μ) be a homogeneous space endowed with a measure μ and a uniformly doubling metric d (that is, $\mu(B(x, 2r)) \leq C\mu(B(y, R))$ for every $x, y \in X$ and $r > 0$). Assume further that (X, d, μ) has a bounded geometry, that is, for some $b > 0$, every $x, y, z \in X$ and every $t \in [\frac{1}{2}, 1]$, the estimate

$$d(y, z) \leq \mathrm{bd}\,(\Phi(x, y, t), \Phi(x, z, t))$$

holds. Here, Φ is a function from $X \times X \times [0, 1]$ to X satisfying

$$\Phi(x, y, s) = \Phi(y, x, 1 - s) = \gamma_{x,y}(s),$$

where $\gamma_{x,y}$ is an appropriate geodesic with values in X. The author proves that, for such a space, the weak Poincaré inequality holds. More precisely, we have, for every measurable function $u \in X$ and g its upper gradient in the sense of J. Heinonen and P. Koskela [Acta Math. **181** (1998), no. 1, 1–61; MR1654771],

$$\int_{B(a,R)} \int_{B(a,R)} |u(x) - u(y)|\, d\mu(y) \leq MR \int_{B(a,3R)} g(z)\, d\mu(z),$$

where $a \in X$, $R > 0$, and M is a constant which depends on b and the uniform doubling constant of X. The proof is based on a generalized change of variables formula.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.