MR1860867 (2002g:53068) 53C24 53C42
Ranjbar-Motlagh, Alireza (BR-FMG)
Rigidity of spheres in Riemannian manifolds and a non-embedding theorem.
(English summary)
Bol. Soc. Brasil. Mat. (N.S.) **32** (2001), *no. 2,* 159–171.

Let $f: M \to \overline{M}$ be an isometric immersion between Riemannian manifolds. The aim of this paper is to find the minimum possible conditions on M and $\overline{M}$ (in terms of curvatures and external diameter) such that the image of f is contained in a geodesic sphere. The author proves the following pinching theorem: Let $f: M^n \to \overline{M}^{n+k}$ be an isometric C^2 immersion. Denote the l-mean curvature vector of $f(M)$ in $\overline{M}$ by H_l. Suppose that the image of f is contained in $B(p, R) \smallsetminus C_p$, where $B(p, R)$ denotes the closed ball in $\overline{M}$ centered at p with radius R and C_p is the cut locus of p, and there is a point $x_0 \in M$ such that $f(x_0) \in \partial B(p, R)$. Suppose that the Hessian of the distance function on $\overline{M}$, $r(y) = r_p(y)$, is bounded from below by $m(r) > 0$ on the tangent bundle of $\partial B(p, r)$. Moreover, suppose that for all $x \in M$ and any unit normal vector field $\eta(f(x))$ on M, we have $|\langle H_l(f(x)), \eta(f(x)) \rangle| \leq m(r(f(x)))|\langle H_{l-1}(f(x)), \eta(f(x)) \rangle|$, where $l \geq 1$ is an integer (define $|\langle H_0(f(x)), \eta(f(x)) \rangle| := 1$). If M is connected, then the image of f is contained in the geodesic sphere $\partial B(p, R)$.

The result generalizes previous results by F. Fontenele and S. L. Silva [Arch. Math. (Basel) **73** (1999), no. 6, 474–480; MR1725184], S. Markvorsen [Math. Z. **183** (1983), no. 3, 407–411; MR0706398] and the reviewer [Geom. Dedicata **68** (1997), no. 1, 73–78; MR1485385]. In addition, the author proves a non-embedding result that extends well-known results of S. Chern and N. H. Kuiper [Ann. of Math. (2) **56** (1952), 422–430; MR0050962] and J. D. Moore [Duke Math. J. **42** (1975), 191–193; MR0377776]. The proofs of the results are based on the maximum principle. *Theodoros Vlachos*

References

1. Yu.E. Borovskii and S. Z. Shefel', *One Chern-Kuiper Theorem*, Siberian Math. J., **19(6):** (1979), 978–979.
2. Yu. Burago, V. Zalgaller, *Geometric Inequalities*, Springer-Verlag, New York, (1988). MR0936419
3. M. P. do Carmo, *Differentiable Curves and Surfaces*, Prentice-Hall, New Jersey, (1976).
4. M. P. do Carmo, *Geometria Riemanniana*, IMPA, Rio de Janeiro, (1988). MR0651516
5. J. Cheeger and D. Ebin, *Comparison Theorems in Riemannian Geometry*, North-Holland Publishing Co., New York, (1975). MR0458335
6. B. Y. Chen, *Geometry of Submanifolds*, Marcel Dekker, New York, (1973). MR0353212
7. S. Y. Cheng, *Eigenvalue Comparison Theorems and Its Geometric Applications*, Math. Z., **143:** (1975), 289–297. MR0378001
8. S. S. Chern and N. Kuiper, *Some Theorems on the Isometric Embedding of Compact Riemannian Manifolds in Euclidean Spaces*, Ann. Math., **56:** (1952), 422–430. MR0050962
9. L. Coghlan and Y. Itokawa, *On the Sectional Curvature of Compact Hypersurfaces*,

Proc. Amer. Math. Soc., **109(1):** (1990), 215–221. MR1010797

10. M. Dajzer, *Submanifolds and Isometric Immersions*, Math. Lect. Ser., Vol. 13, Publish or Perish Inc, Houston, (1990). MR1075013

11. F. Fontenele and S. L. Silva, *On the Scalar Curvature of Compact Hypersurfaces*, Arch. Math., **73:** (1999), 474–480. MR1725184

12. D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Second Edition, Springer-Verlag, New York, (1983). MR0737190

13. M. Gromov, *Partial Differential Relations*, Springer-Verlag, New York, (1986). MR0864505

14. N. J. Hicks, *Notes on Differential Geometry*, D. Van Nostrand Company Inc, New York, (1965). MR0179691

15. T. Ishihara, *Radii of immersed manifolds and non-existence of immersions*, Proc. Amer. Math. Soc., (1980), pp. 276–279. MR0550512

16. H. Jacobowitz, *Isometric Embedding of a Compact Riemannian Manifold into Euclidean Space*, Proc. Amer. Math. Soc., **40:** (1973), 245–246. MR0375173

17. L. Jorge and D. Koutroufiotis, *An Estimate for the Curvature of Bounded Submanifolds*, Amer. J. Math., **103(4):** (1981), 711–725. MR0623135

18. L. Jorge and F. V. Xavier, *An Inequality between the Exterior Diameter and the Mean Curvature of Bounded Immersions*, Math. Z., **178:** (1981), 77–82. MR0627095

19. D. Koutroufiotis, *Elementary Geometric Applications of a Maximum Principle for Nonlinear Elliptic Operators*, Arch. Math., **24:** (1973), 97–99. MR0315635

20. S. Markvorsen, *A Sufficient Condition for a Compact Immersion to be Spherical*, Math. Z., **183:** (1983), 407–411. MR0706398

21. J. D. Moore, *An Application of Second Variation to Submanifold Theory*, Duke Math. J., **42:** (1975), 191–193. MR0377776

22. P. F. Leung, *On the Ricci Curvature of a Compact Hypersurface in Euclidean Space*, Ann. of Global Anal. Geom., **13:** (1995), 55–58. MR1327111

23. L. Rodríguez, *Geometria das Subvariedades*, Monografias de Matemática, Vol. 26, IMPA, Rio de Janeiro, (1976).

24. R. M. Schoen and S. T. Yau, *Lectures on Differential Geometry*, Vol. I, International Press, (1994). MR1333601

25. T. Vlachos, *A Characterization for Geodesic Spheres in Space Forms*, Geom. Dedicata, **68:** (1997), 73–78. MR1485385

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.